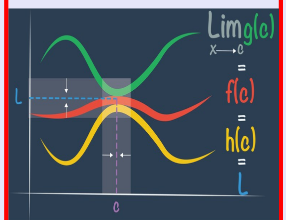
Math 261

Spring 2021

Lecture 22



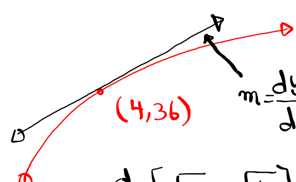
Find eqn of the tan. line to the curve given by $\sqrt{x} + \sqrt{y} = 8$ at $(4,36)$

Verify the Point:

$\sqrt{4} + \sqrt{36} = 8$

$2 + 6 = 8$

$8 = 8$ ✓



$m = \left.\frac{dy}{dx}\right|_{(4,36)}$

$$\frac{d}{dx}\left[\sqrt{x} + \sqrt{y}\right] = \frac{d}{dx}[8]$$

$$\frac{d}{dx}\left[x^{1/2} + y^{1/2}\right] = 0$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$

multiply by 2, isolate $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

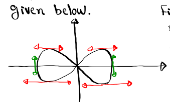
$$m = \left.\frac{dy}{dx}\right|_{(4,36)} = -\frac{\sqrt{36}}{\sqrt{4}} = \boxed{-3}$$

Now tan. line $y - 36 = -3(x - 4)$

$\vdots$

$\boxed{y = \quad}$

Graph of $8(x^2+y^2)^2 = 100(x^2-y^2)$ is given below.



Find x-values of the Points where we have horizontal tan. lines.

$m=0$

$\frac{dy}{dx}=0$

$$\frac{d}{dx}\left[8(x^2+y^2)^2\right] = \frac{d}{dx}\left[100(x^2-y^2)\right]$$

$$8 \cdot 2(x^2+y^2)^1 \cdot \left(2x + 2y \cdot \frac{dy}{dx}\right) = 100\left(2x - 2y \cdot \frac{dy}{dx}\right)$$

$$32(x^2+y^2)\left(x + y\frac{dy}{dx}\right) = 200\left(x - y\frac{dy}{dx}\right)$$

$$32(x^2+y^2)x + 32(x^2+y^2) \cdot y\frac{dy}{dx} = 200x - 200y\frac{dy}{dx}$$

$$32(x^2+y^2)y\frac{dy}{dx} + 200y\frac{dy}{dx} = 200x - 32(x^2+y^2)x$$

$$\left(32y(x^2+y^2) + 200y\right)\frac{dy}{dx} = 200x - 32x(x^2+y^2)$$

$$\frac{dy}{dx} = \frac{200x - 32x(x^2+y^2)}{32y(x^2+y^2) + 200y}$$

For Horizontal tan. lines $\frac{dy}{dx}=0$

$200x - 32x(x^2+y^2) = 0$ As long as Deno. $\neq 0$.

$200x - 32x(x^2+y^2) = 0$

$x\left(200 - 32(x^2+y^2)\right) = 0$

$x=0 \qquad 200 - 32(x^2+y^2) = 0$

$$\begin{cases} 200 - 32(x^2+y^2) = 0 \\ 8(x^2+y^2)^2 = 100(x^2-y^2) \end{cases} \Rightarrow x^2+y^2 = \frac{200}{32} = \frac{50}{8} = \frac{25}{4}$$

$$8\left(\frac{25}{4}\right)^2 = 100(x^2-y^2)$$

$$\cancel{8} \cdot \frac{625}{\cancel{16}_{2}} = 100(x^2-y^2) \Rightarrow x^2-y^2 = \frac{625}{200} = \frac{\cancel{25} \cdot 25}{\cancel{25} \cdot 8}$$

$$\begin{cases} x^2-y^2 = \frac{25}{8} \\ x^2+y^2 = \frac{25}{4} \end{cases}$$

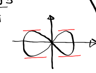
$2x^2 = \frac{25}{8} + \frac{50}{8}$

$2x^2 = \frac{75}{8}$

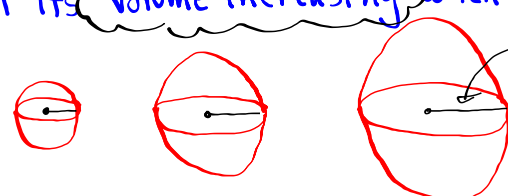
$x^2 = 75/16$

$x = \pm\sqrt{\frac{75}{16}}$

$x = \pm\frac{5\sqrt{3}}{4}$



Salma is blowing air into a basketball. Radius is increasing at the rate of 2 cm/sec. How fast its volume increasing when radius is 6 cm?



$\frac{dr}{dt} = 2$ cm/sec.

$\frac{dV}{dt} = ?$ when $r = 6$ cm.

Volume of Sphere $V = \frac{4\pi r^3}{3}$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

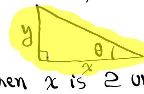
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(6)^2 \cdot 2$$

$$\boxed{\frac{dV}{dt} = 288\pi \text{ cm}^3/\text{sec}}$$

March 18, 2021

Consider a right triangle with angle θ and x is the adjacent side, and y is opposite Side.



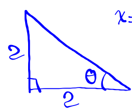
At certain time, when x is 2 units and it is increasing at 1 unit/Sec. $\frac{dx}{dt} = 1$ unit/Sec
y is also 2 unit and it is decreasing at $\frac{1}{4}$ unit/ Sec. $\frac{dy}{dt} = -\frac{1}{4}$ unit/Sec.

How fast the angle θ changing? $\left.\frac{d\theta}{dt}\right|_{\substack{x=2\\y=2}}$?

Is θ increasing or decreasing? why?

$\tan\theta = \frac{y}{x}$

$\frac{d}{dt}[\tan\theta] = \frac{d}{dt}\left[\frac{y}{x}\right]$



$x=2$
$y=2$
$\theta = 45^\circ$
$\theta = \frac{\pi}{4}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot x - y\cdot\frac{dx}{dt}}{x^2}$

when $x=2$ $y=2$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sec\frac{\pi}{4} = \sqrt{2}$ $\sec^2\frac{\pi}{4} = 2$

$\sec^2\frac{\pi}{4}\,\frac{d\theta}{dt} = \frac{-\frac{1}{4}\cdot 2 - 2\cdot 1}{2^2}$

$2\cdot\frac{d\theta}{dt} =$

$\frac{d\theta}{dt} = -$ Radians/Sec

Decreasing

Class QZ 9

Find all x-values for which the tangent to $f(x) = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 10$.